The $\mathcal{M}_{gg\to gH}$ amplitude at one loop with pySecDec

ACAT 2021 : 20th International Workshop on Advanced Computing and Analysis Techniques in Physics Research

November 29, 2021 to December 3, 2021
Virtual and IBS Science Culture Center, Daejeon, South Korea

Collaboration of . . .

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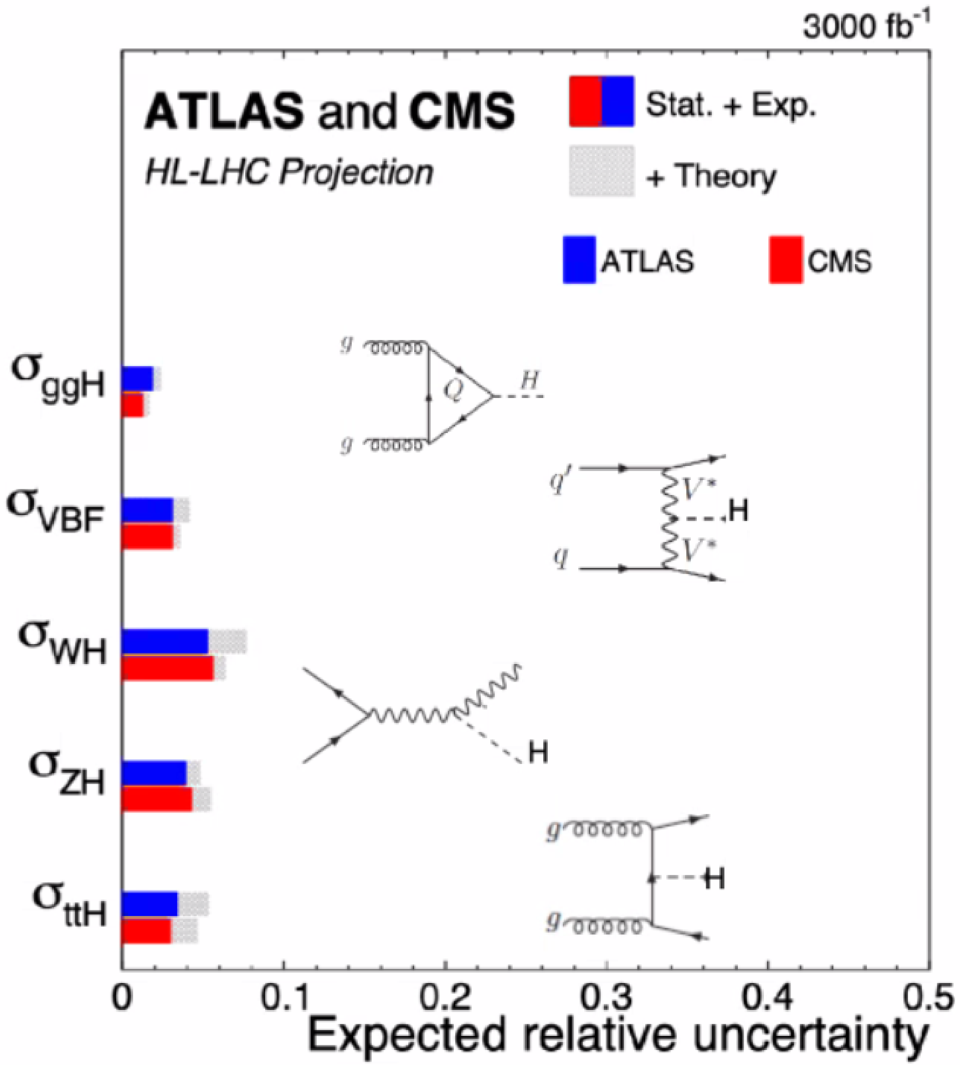
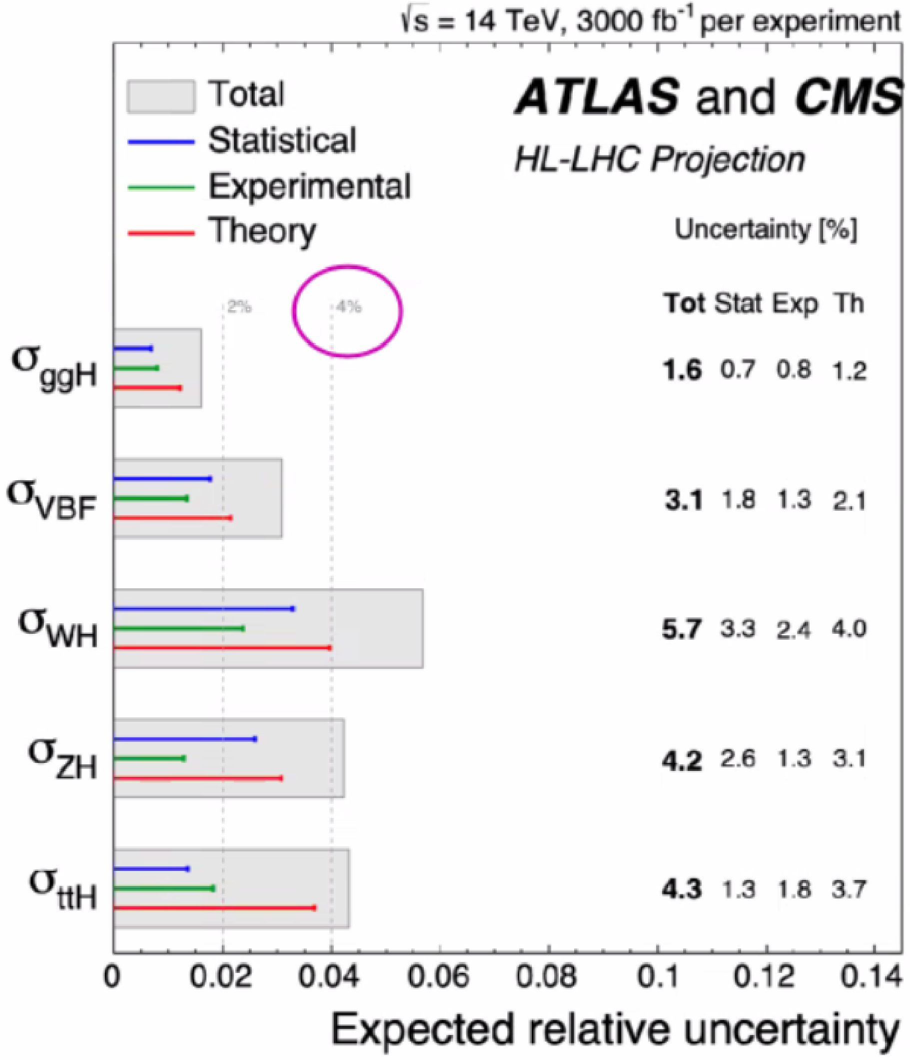
1.

Higgs production at HL-LHC: expected precision

~ From a talk by G.H.





HL-HE CERN Yellow Report, 1902.00134

2.

Role of pySecDec

Theoretical precision → Higher order corrections → Multi-loop integrals

pySecDec : Numerical evaluation of dimensionally regulated parameter integrals using sector decomposition*

→ We will introduce two features that will be helpful for Multi-loop calculations

* Sophia Borowka, Gudrun Heinrich, Stephan Jahn, Stephen Jones, Matthias Kerner, Florian Langer, Vitaly Magerya, Andres Poldaru, Johannes Schlenk, Emilio Villa, Tom Zirke

GitHub : github.com/gudrunhe/secdec

ReadTheDocs : secdec.readthedocs.io/

3.

Two major new features of pySecDec

Direct Numerical evaluation of weighted sum of loop integrals, with the new **"sum_package"**

- Optimised to reach given accuracy goal on the sum rather than on individual integrals
- Quantities like F ~ Form factor can be directly evaluated :

$$F = c_1 I_1 + c_2 I_2 + \cdots$$

Introduced to pySecDec in arXiv:2108.10807

4.

Two major new features of pySecDec

➡ Evaluation of loop integrals with "Expansion by regions" * method

Integrals expressed as series expansion in terms of a 'smallness parameter'

- Ideal to apply when large scale differences are present between the invariants of the Integral

Hard region : k ~ p >> m $\qquad \frac{1}{(k+p)^2(k^2-m^2)^2} \to \frac{1}{(k+p)^2\,(k^2)^2}\left(1+2\frac{m^2}{k^2}+\ldots\right)\,,$

Soft region : k ~ m << p $\qquad \frac{1}{(k+p)^2(k^2-m^2)^2} \to \frac{1}{(k^2-m^2)^2\,p^2}\left(1-2\frac{p\cdot k}{p^2}-\frac{k^2}{p^2}+\ldots\right)$

Introduced to pySecDec in arXiv:2108.10807

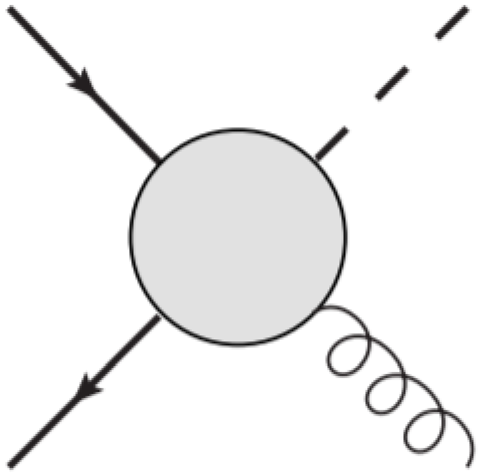
* hep-ph/9711391

5.

Let's study the application of both of these new features to Higgs plus jet production at one loop

Various partonic channels contribute to Higgs+jet production amplitude

$$qg \to qH, \quad gq \to qH, \quad \bar{q}g \to \bar{q}H, \quad g\bar{q} \to \bar{q}H,$$
$$q\bar{q} \to gH, \quad \bar{q}q \to gH, \quad gg \to gH.$$



- However, the calculation methodology is similar for all processes, thus we focus on the g g → g H process.

6.

The $\mathcal{M}_{gg\to gH}$ amplitude at one loop

Helicity amplitudes → Form factors → Master integrals

- Our task would be to express each physical form factor as linear combination of the '14 Integrals'. Thereby making it ready to evaluate with the "sum_package".

$$F = \sum P_n I_n$$

- The 14 integrals consist of 9 master integrals and 5 crossed integrals. Let us quickly review these integrals.

Complete analytical treatment in arXiv:1711.09875

7.

The Master Integrals

- **General scalar Feynman integral :**

$$s_{ij} = (p_i + p_j)^2$$

$$p_1^2 = p_2^2 = p_3^2 = 0 \quad \& \quad p_4^2 = m_H^2$$

$$p_1 + p_2 + p_3 + p_4 = 0 \;\Rightarrow\; m_H^2 = s_{12} + s_{13} + s_{23}$$

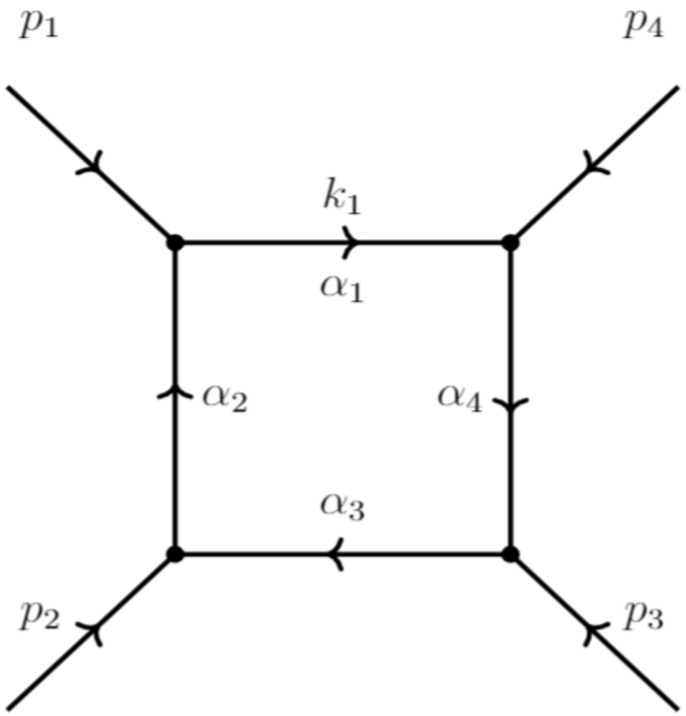
$$I_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} = \int \frac{d^d k_1}{i\pi^{d/2}} \cdot \frac{1}{D_1^{\alpha_1} D_2^{\alpha_2} D_3^{\alpha_3} D_4^{\alpha_4}}$$

$$D_1 = k_1^2 - m_t^2$$

$$D_2 = (k_1 - p_1)^2 - m_t^2$$

$$D_3 = (k_1 - p_1 - p_2)^2 - m_t^2$$

$$D_4 = (k_1 - p_1 - p_2 - p_3)^2 - m_t^2$$



- Different sub-topologies obtained by pinching various propagators, i.e. setting $\alpha_i = 0$

8.

The Form Factors

➡ There are 4 physical form factors : F_{212} , F_{311}, F_{332}, F_{312}

- Because of the permutation invariance of the amplitude :

$$F_{212}(s_{12},s_{13},s_{23}) = F_{311}(s_{13},s_{23},s_{12}) = F_{332}(s_{23},s_{12},s_{13})$$

- Thus only two of them independent. We take them to be F_{212} , F_{312} .

  Calculate them in terms of linear combinations of 14 master integrals

$$F_{212} = \sum_{n=1}^{14} (P_n I_n) \qquad F_{312} = \sum_{n=1}^{14} (M_n I_n)$$

9. Helicity amplitudes

➡ Each gluon can have two helicities, we have $2^3 = 8$ helicity amplitudes

- Only two of them are independent, as a result of parity invariance and permutation invariance of the amplitude.

$$\mathcal{M}_{gg\to gH}^{h_1 h_2 h_3} = -\mathcal{M}_{gg\to gH}^{-h_1 -h_2 -h_3}$$

$$\mathcal{M}_{gg\to gH}^{++-}(s_{12}, s_{13}, s_{23}) = \mathcal{M}_{gg\to gH}^{+-+}(s_{13}, s_{23}, s_{12}) = \mathcal{M}_{gg\to gH}^{-++}(s_{23}, s_{12}, s_{13})$$

- Amplitude squared in terms of helicity amplitudes (We use this equation for final numerical evaluation)

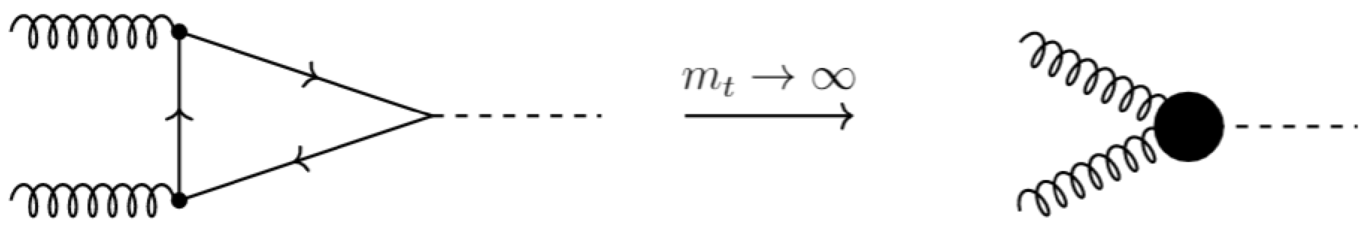
$$|\mathcal{M}_{gg\to gH}|^2 = 2\cdot\left(|\mathcal{M}_{gg\to gH}^{+++}|^2 + |\mathcal{M}_{gg\to gH}^{++-}|^2 + |\mathcal{M}_{gg\to gH}^{+-+}|^2 + |\mathcal{M}_{gg\to gH}^{-++}|^2\right)$$

10. Analytic expressions

The **analytic expression** for **amplitude squared** in terms of Lorentz invariants.

$$m_t \to \infty \Rightarrow m_t^2 >> |s_{ij}| \ \& \ m_t^2 >> m_H^2$$

(This is the **analytic** result in **Heavy Top Limit**)



$$|\mathcal{M}_{gg\to gH}|^2 = \frac{4}{9m_t^4} \cdot \frac{(m_H^8+s_{12}^4+s_{13}^4+s_{23}^4)}{s_{12}s_{13}s_{23}} + \mathcal{O}(\epsilon)$$

11.

Implementation in pySecDec

- First, define all the 14 integrals and corresponding coefficients in a separate file. (One for each form factor)
- Invoke the sum_package to evaluate the weighted sum. Define necessary parameters for integration and relative accuracy.
- For our example ~ weighted sum of 14 one-loop integrals,
- CPU @ 1.6 GHz : Relative precision ~ 10^{-4}
  computing time ~ 10 s

More information ~ Monday 29/11 : "Expansion by regions & Monte Carlo integration with pySecDec" ~ Dr. Vitalii Maheria

12.

Results

Integration conditions : $m_t^2 >> |s_{ij}| \; \& \; m_t^2 >> m_H^2$

- $[s_{12}, s_{13}, s_{23}, m^2_t, m^2_H] = [0.0009, -0.0003, -0.000442, 1.0, 0.000157]$
- Integrator = Qmc   verbose = True   epsrel = 1e-4   epsabs = 1e-14

Integration results!

Numerical Result of The Ampltiude Squared with Helicity amplitudes

$|M(gg \to gH)|^2$: (2.61402286630 +/- 0.00000000264) $* \; 10^{-3}$

Analytic Result of The Ampltiude Squared in Heavy Top Limit

$|M(gg \to gH)|^2$: (2.61397604127) $* \; 10^{-3}$

% Difference with respect to analytic result in HTL : 0.00179

13.

Expansion by regions

➡ Express the integral as series expansion in 'smallness parameter' ~ z

- For the master integrals of our amplitude, we can expand them in inverse powers of m^2_t, considering we will evaluate them in heavy top limit

- First order expansion for I_2 integral is expressed :

$$g_2(s_{12}, m_t^2) = \frac{-(6m_t^2 + s_{12})}{12m_t^4} + \epsilon \cdot \left(\frac{\gamma_E(6m_t^2 + s_{12})}{12m_t^4} + \frac{-(s_{12} + (6m_t^2 + s_{12}) \cdot \ln \frac{1}{m_t^2})}{12m_t^4} \right)$$

- Define loop integral regularly, but multiply all invariants and masses (except m_t) with 'smallness parameter' ~ z .

14.

Results

Integration conditions :

$$m_t^2 >> |s_{ij}| \ \& \ m_t^2 >> m_H^2$$

- $[s_{12}, s_{13}, s_{23}, m^2_t, m^2_H] = [0.0009, -0.0003, -0.000442, 1.0, 0.000157]$
- Integrator = Qmc Z = 1 expansion_by_regions_order = 1

Integration results! (Only for the I_2 integral, others can be verified similarly)

```
Analytic Result :
eps^0 :  -0.500074999999999
eps^1 :   0.288576123625634
Numerical Result :
eps^0 :  -0.500075000000000+0.000000000000000*I
          +/- 3.389926606603533e-17+1.358594763071333e-17*I
eps^1 :   0.288576123625634-0.000000000000000*I
          +/- 1.956787297766954e-17+7.843700577440028e-18*I
```

Summary & Outlook

- Latest version : pySecDec 1.5.2 (arXiv:2108.10807)
- Demonstrated the application of two new features with three gluon Higgs amplitude at 1-loop ~ Higgs+jet production through ggF
  - Direct numerical evaluation of weighted sums of master integrals
  - Numerical evaluation of loop Integrals with method of expansion by regions
- Efficient tools to tackle Multi-loop calculations

Thank you for listening!

Extra slides

The Master Integrals

- **The final 14 Integrals are as follows. We maintain the same convention of order throughout this project.**

$$I_1 = \quad g_1(m_t^2) = I_{2,0,0,0}$$

$$I_2 = \quad g_2(s_{12}, m_t^2) = I_{2,0,1,0}$$

$$I_3 = \quad g_2(s_{13}, m_t^2) = I_{2,0,1,0} \quad : (p_2 \leftrightarrow p_3)$$

$$I_4 = \quad g_3(s_{23}, m_t^2) = I_{0,2,0,1}$$

$$I_5 = \quad g_4(m_H^2, m_t^2) = I_{2,0,0,1}$$

$$I_6 = \quad g_5(s_{12}, m_t^2) = I_{1,1,1,0}$$

$$I_7 = \quad g_5(s_{13}, m_t^2) = I_{1,1,1,0} \quad : (p_2 \leftrightarrow p_3)$$

$$I_8 = \quad g_6(s_{23}, m_t^2) = I_{0,1,1,1}$$

$$I_9 = \quad g_7(s_{12}, m_H^2, m_t^2) = I_{1,0,1,1}$$

$$I_{10} = \quad g_7(s_{13}, m_H^2, m_t^2) = I_{1,0,1,1} \quad : (p_2 \leftrightarrow p_3)$$

$$I_{11} = \quad g_8(s_{23}, m_H^2, m_t^2) = I_{1,1,0,1}$$

$$I_{12} = \quad g_9(s_{12}, s_{23}, m_H^2, m_t^2) = I_{1,1,1,1}$$

$$I_{13} = \quad g_9(s_{12}, s_{13}, m_H^2, m_t^2) = I_{1,1,1,1} \quad : (p_1 \leftrightarrow p_2)$$

$$I_{14} = \quad g_9(s_{23}, s_{13}, m_H^2, m_t^2) = I_{1,1,1,1} \quad : (p_2 \leftrightarrow p_3)$$

The $g_5(s_{12}, m_t^2)$ integral

- This integral needs an additional regulator while evaluating with expansion by regions. (pySecDec will notify)

- Introduce the extra regulator n_1 in the powers of the propagators, by consequently dividing it with prime numbers.

- Modified conditions for I_6 integral are as follows : (remaining conditions similar to others)

```
powerlist = ["1+n1", "1+ n1/2", "1+n1/3", "0+n1/5"]

regulators=["n1","eps"]  requested_orders = [0,1]
```

Error analysis

Stays constant : $\frac{|s_{ij}|}{m_t^2} \sim \frac{m_H^2}{m_t^2} \sim 10^{-4}$

m_t^2	$\mathcal{A}$	$\Delta\mathcal{A}$	$\mathcal{A}_0$	% Numerical error	% Deivation
10^{-4}	$2.61402286225 \cdot 10^{1}$	$1.17774107579 \cdot 10^{-8}$	$2.61397604127 \cdot 10^{1}$	$4.505473509 \cdot 10^{-8}$	$1.79117853416 \cdot 10^{-3}$
10^{-2}	$2.61402286406 \cdot 10^{-1}$	$1.02678005700 \cdot 10^{-10}$	$2.61397604127 \cdot 10^{-1}$	$3.9279689215 \cdot 10^{-8}$	$1.79124787455 \cdot 10^{-3}$
1	$2.61402286630 \cdot 10^{-3}$	$2.64465133233 \cdot 10^{-12}$	$2.61397604127 \cdot 10^{-3}$	$1.0117169847 \cdot 10^{-7}$	$1.79133366574 \cdot 10^{-3}$
10^{2}	$2.61402287068 \cdot 10^{-5}$	$2.75310779611 \cdot 10^{-14}$	$2.61397604127 \cdot 10^{-5}$	$1.0532072336 \cdot 10^{-7}$	$1.79150118589 \cdot 10^{-3}$
10^{4}	$2.61402285788 \cdot 10^{-7}$	$2.82184625032 \cdot 10^{-16}$	$2.61397604127 \cdot 10^{-7}$	$1.0795032805 \cdot 10^{-7}$	$1.79101118864 \cdot 10^{-3}$
10^{6}	$2.61400648697 \cdot 10^{-9}$	$2.12661964630 \cdot 10^{-14}$	$2.61397604127 \cdot 10^{-9}$	$8.13547960536 \cdot 10^{-4}$	$1.16472764313 \cdot 10^{-3}$
10^{8}	$2.65071422899 \cdot 10^{-11}$	$6.37139264259 \cdot 10^{-13}$	$2.61397604127 \cdot 10^{-11}$	$2.40365127742 \cdot 10^{0}$	$1.40545235067 \cdot 10^{0}$

Table 2: Error analysis with m_t^2 scaling

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